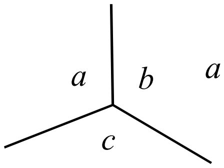
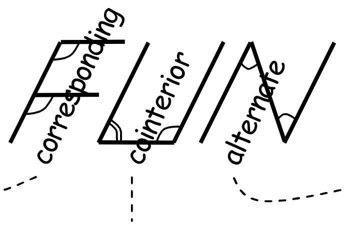
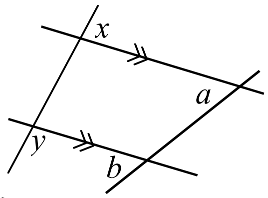
Geometry

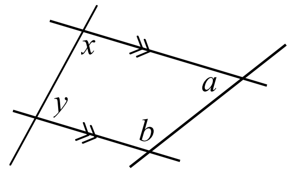


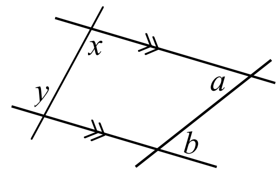
$a+b+c=360^\circ$ ($\angle$'s at a pt)

Parallel lines are

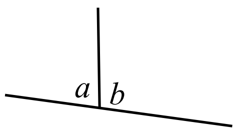




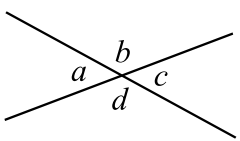




$\left.\begin{array}{l} a=b \\ x=y \end{array}\right\}$ (*corres.*$\angle$'*s*; // lines) $\left.\begin{array}{l} a+b=180^\circ \\ x+y=180^\circ \end{array}\right\}$ (*coint.* $\angle$'s ;// lines) $\left.\begin{array}{l} a=b \\ x=y \end{array}\right\}$ (*alt.*$\angle$'*s*; // lines)



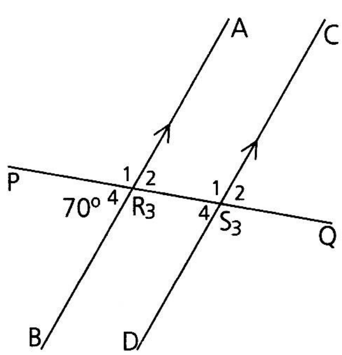
$a+b=180^\circ$ (adj. $\angle$'s st line)



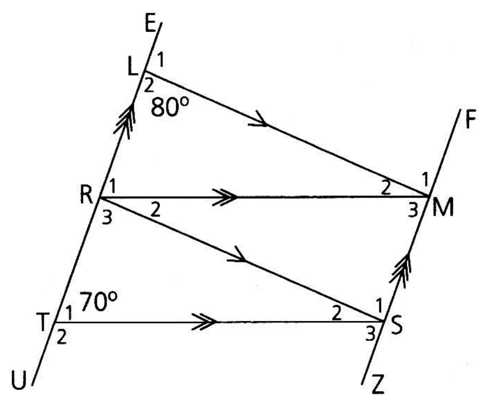
$\left.\begin{array}{l} a=c \\ b=d \end{array}\right\}$ (vert. opp.$\angle$'s)

§ Exercise 1

1.1 If AB ∥ CD and $\hat{R}_4 = 70^\circ$, find the sizes of the other angles, giving reasons.

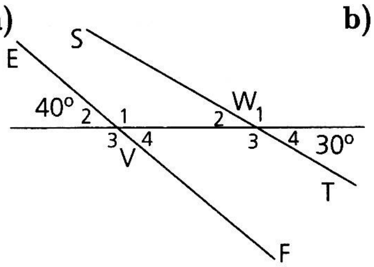


1.2 Determine the sizes of all the unknown angles in this figure.

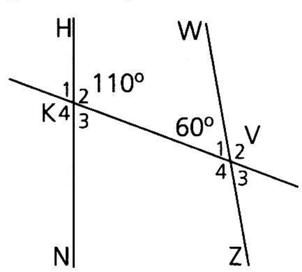


1.3 Determine whether there are pairs of parallel lines in the following figures.

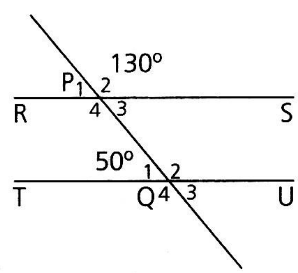
a)



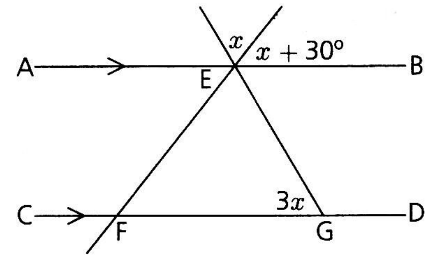
b)



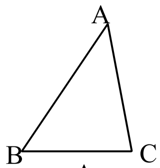
c)



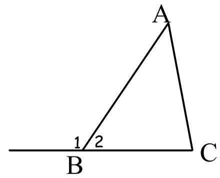
1.4 Calculate x in the figure alongside.



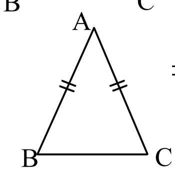
Triangles



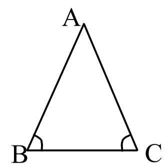
$\hat{A} + \hat{B} + \hat{C} = 180°$ ($\angle$ sum Δ)



$\hat{B}_1 = \hat{A} + \hat{C}$ (ext. $\angle$ Δ)



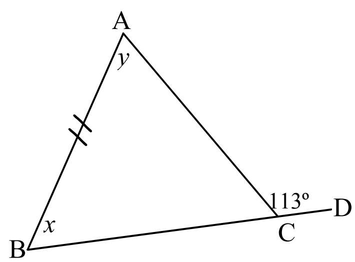
$\Rightarrow \hat{B} = \hat{C}$ (isos. Δ)



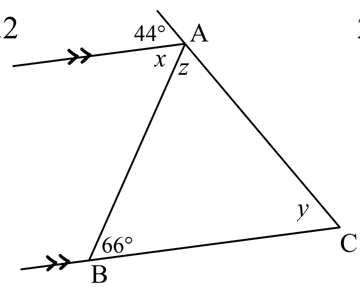
$\Rightarrow AB = AC$ (isos. Δ)

§ Exercise 2 Find the value of x, y and z, giving reasons.

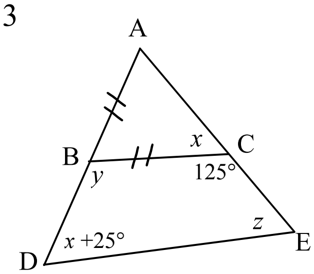
2.1



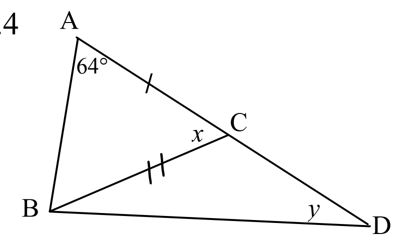
2.2



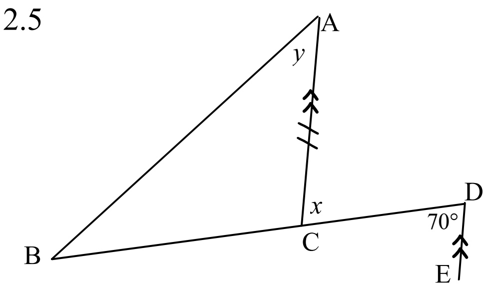
2.3



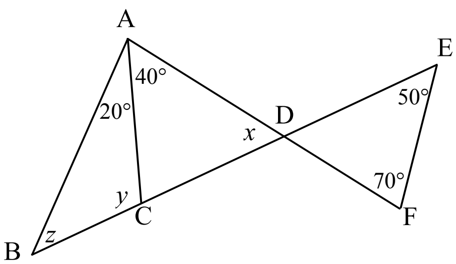
2.4



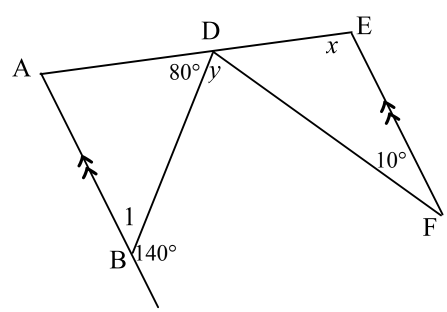
2.5



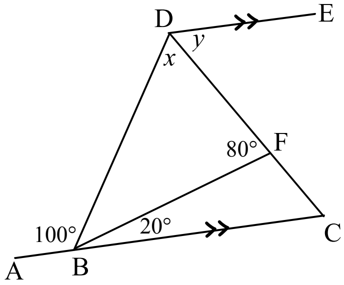
2.6



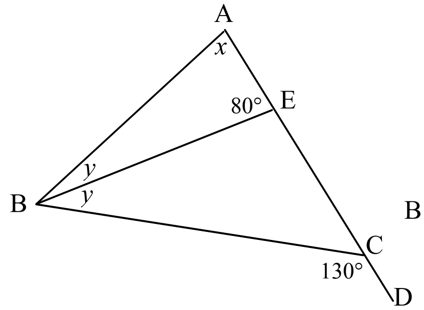
2.7



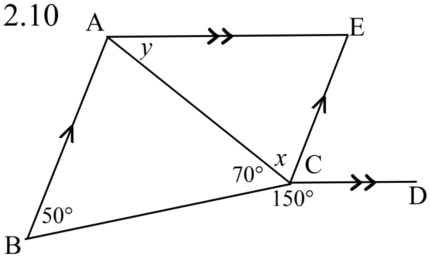
2.8



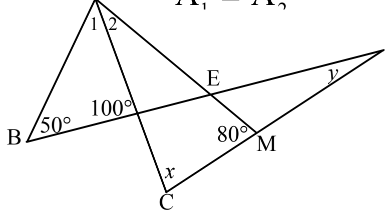
2.9



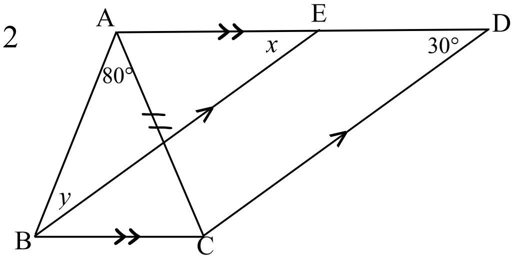
2.10



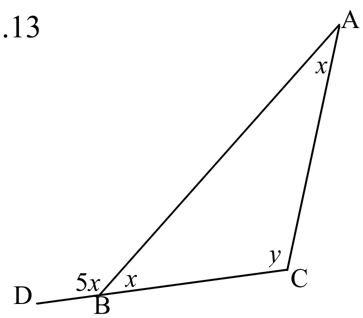
2.11 $\hat{A}_1 = \hat{A}_2$



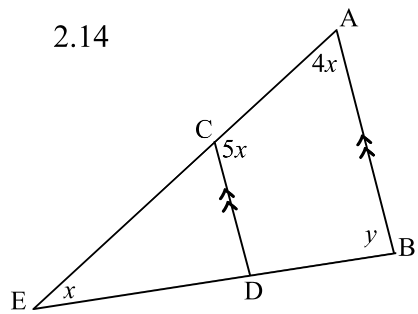
2.12



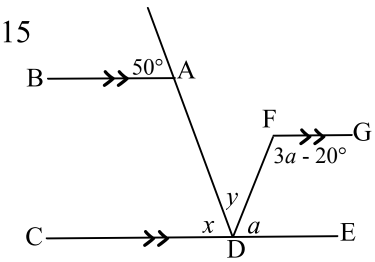
2.13



2.14

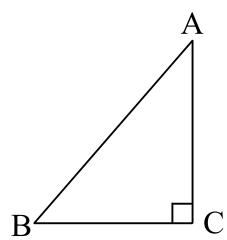


2.15



Pythagoras Theorem

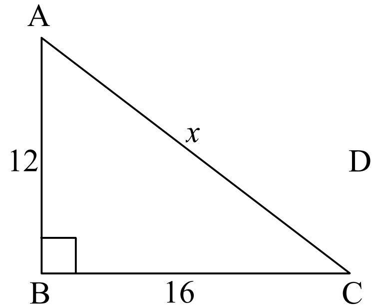
$$AB^2 = AC^2 + BC^2$$



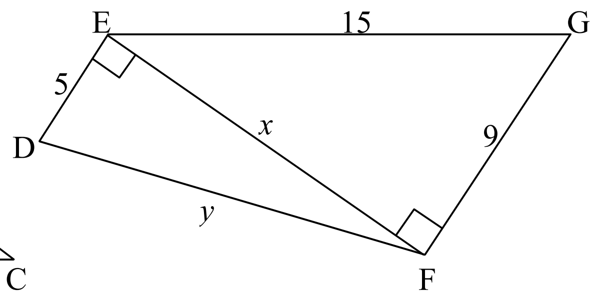
§ Exercise 3

Find the value of *x* and *y*, correct to 2 dec.pl. if necessary:

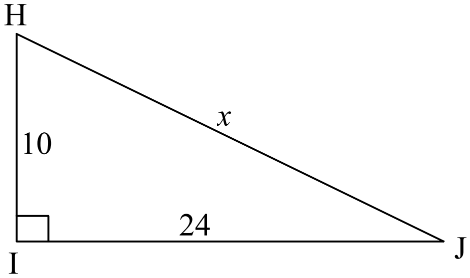
3.1



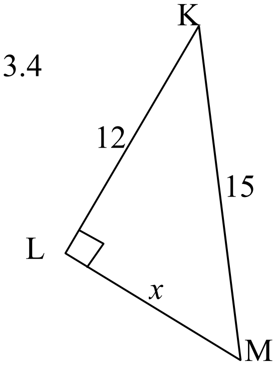
3.2



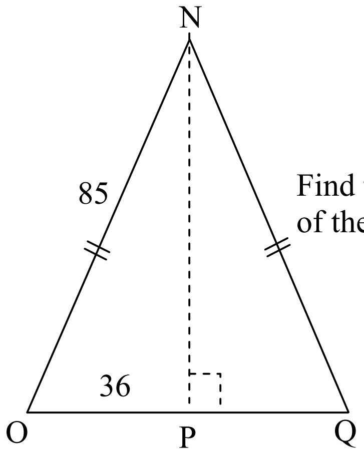
3.3



3.4

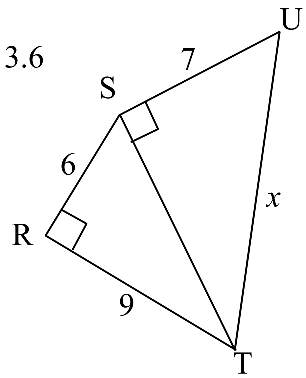


3.5

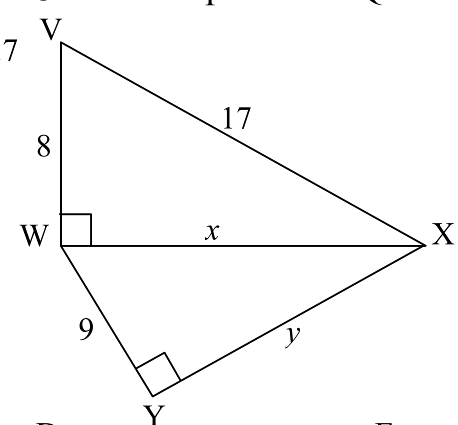


Find the area of the Δ.

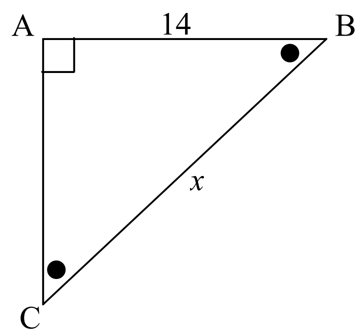
3.6



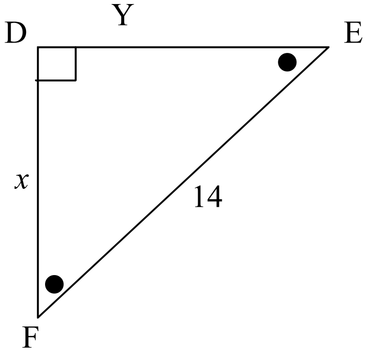
3.7



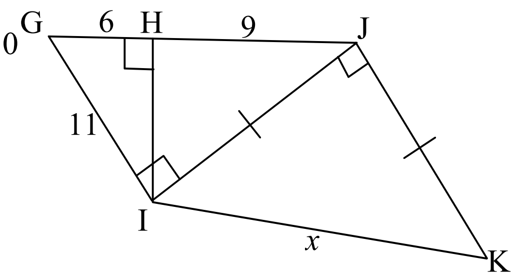
3.8



3.9



3.10



Congruent triangles

i.e. ***exactly the same*** in all respects.

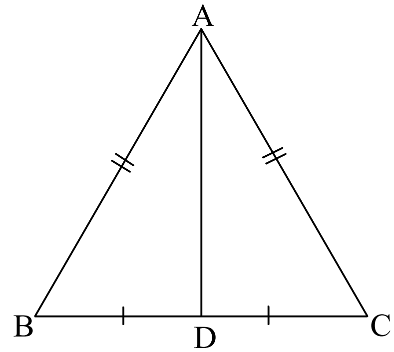
The 4 cases for congruency:

- SSS
- SAA (or AAS or ASA)
- SAS (must be the included angle)
- RHS (right angle, hypotenuse, side)

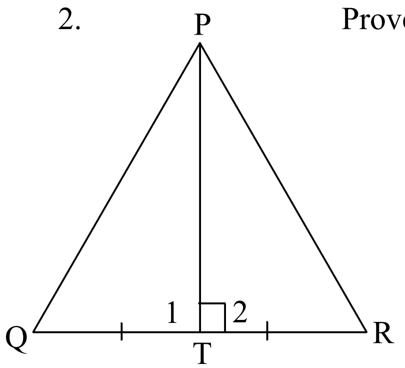
e.g.1 1. In Δ’s ABD, ACD

1. AB = AC (given)
2. BD = DC (given)
3. AD common

$\therefore \Delta ABD \equiv \Delta ACD$ (SSS)



2.



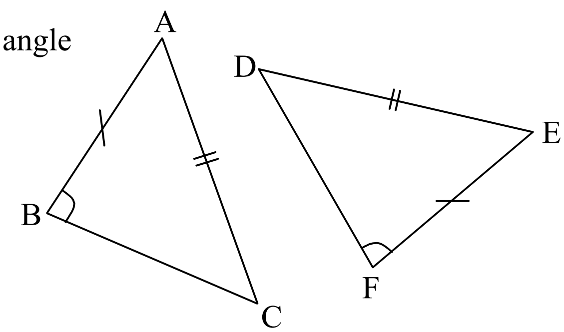
Prove: 2.1 $\Delta PQT \equiv \Delta PRT$
2.2 PQ = PR

» **Solution** 2.1 In Δ’s PQT, PRT
1. QT = TR (given)
2. $\hat{T}_1 = \hat{T}_2 = 90°$ (adj.∠'s st line)
3. PT common

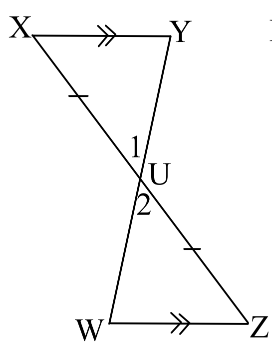
$\therefore \Delta PQT \equiv \Delta PRT$ (SAS)

2.2 $\therefore$ PQ = PR (≡ Δ’s proven)

3. $\Delta ABC \not\equiv \Delta DEF$ because the angle is not included.
(where the sides meet)



4.



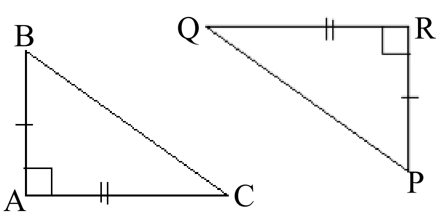
In Δ’s XYU, ZWY:
1. XU = UZ (given)
2. $\hat{U}_1 = \hat{U}_2$ (vert. opp.)
3. $\hat{Y} = \hat{W}$ (alt.∠’s; XY//WZ)

$\therefore \Delta XYU \equiv \Delta ZWU$ (SAA)

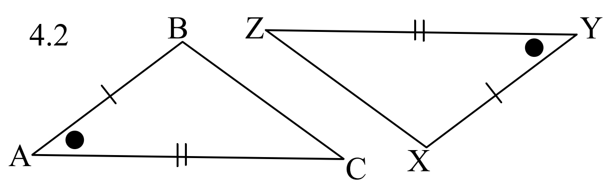
§ Exercise 4

State whether each of the following pairs of triangles are congruent or not. If congruent, state the congruency and the case for congruency.

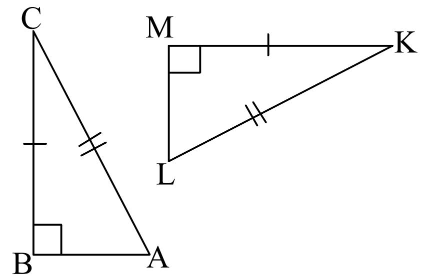
4.1



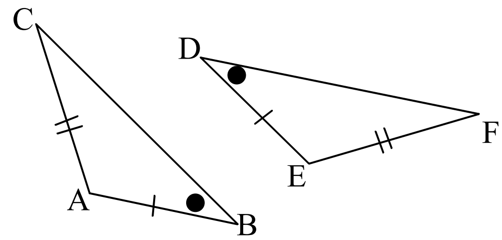
4.2



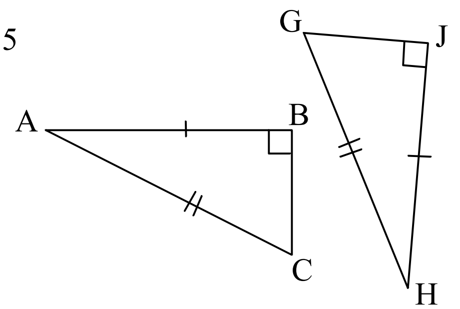
4.3



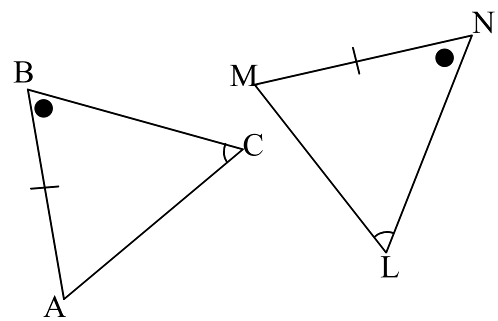
4.4



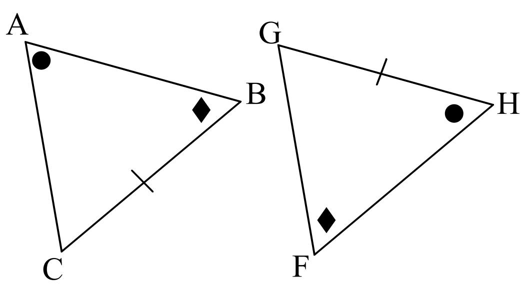
4.5



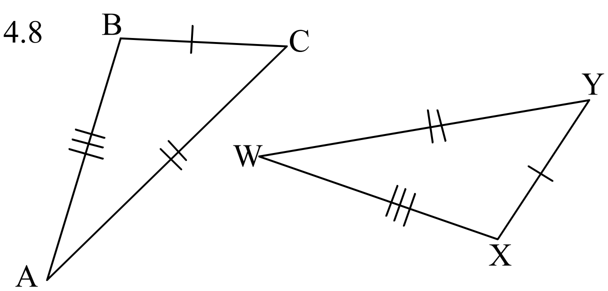
4.6



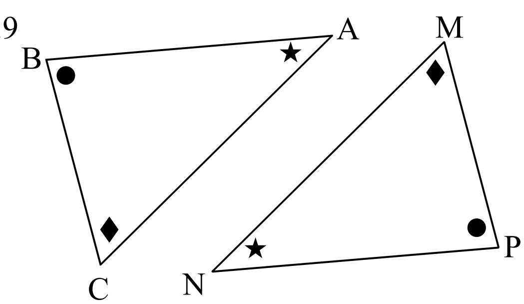
4.7



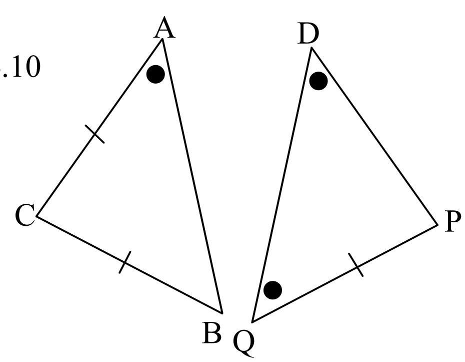
4.8



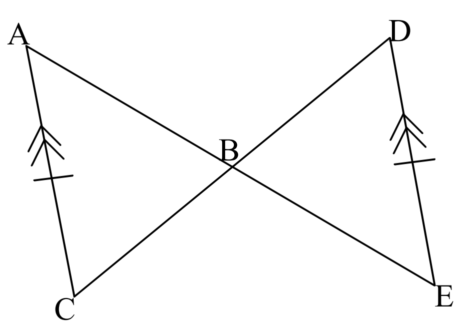
4.9



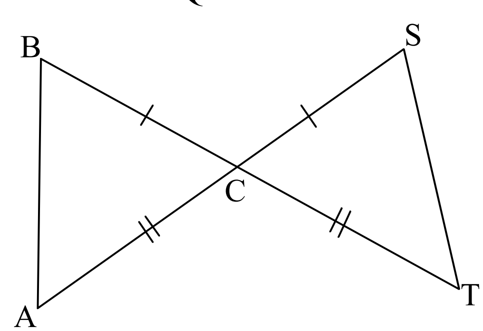
4.10



4.11



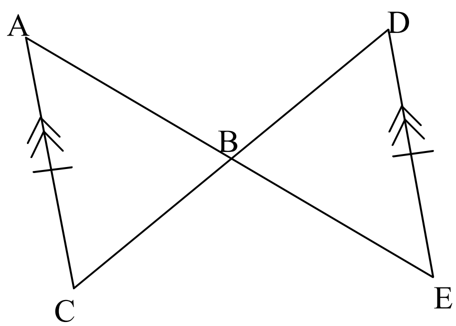
4.12



§ Exercise 5

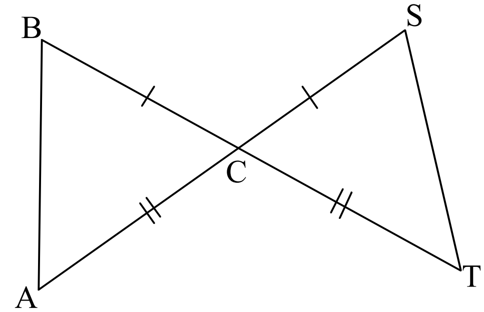
Prove the congruency stated in each of the following:

5.1



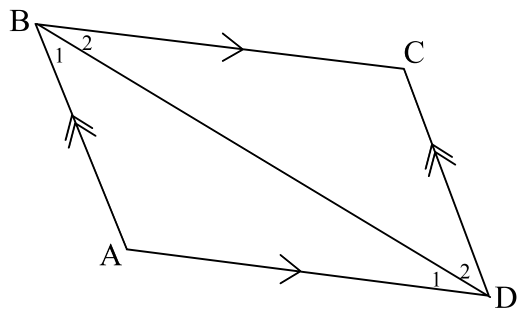
$\Delta ABC \equiv \Delta EBD$

5.2



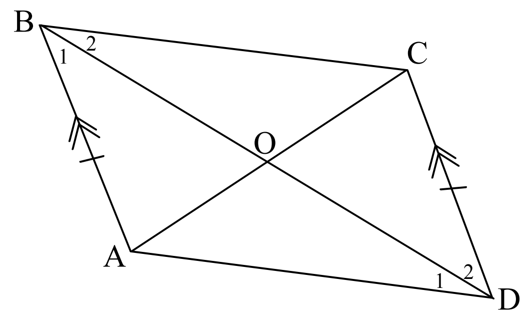
Δ's $ABC \equiv \Delta\ TSC$

5.3



Δ's $ABD \equiv \Delta\ CDB$

5.4

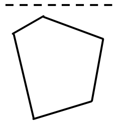


Δ's $ABO \equiv \Delta\ CDO$

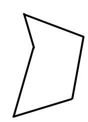
Polygons

No. of sides	
4	quadrilateral
5	pentagon
6	hexagon
7	septagon
8	octagon
9	nonagon
10	decagon

convex e.g.



concave e.g.



"caved in"

regular … all sides equal

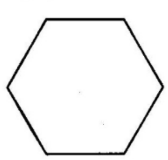
As with triangles, polygons are

- similar if they are the same shape, i.e. equal angles.
- congruent if they are exactly the same in all respects.

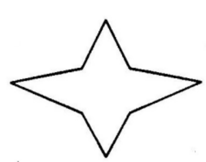
§ Exercise 6

6.1 (i) State whether each of the following polygons is convex or concave.
(ii) Give each polygon its usual name.

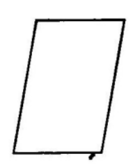
a)



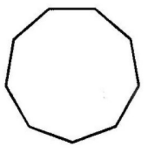
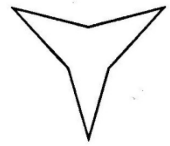
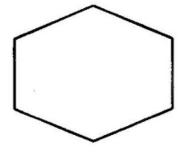
b)



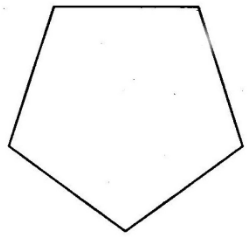
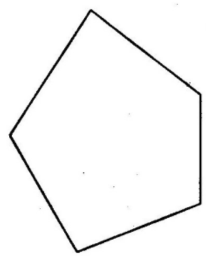
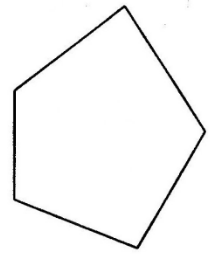
c)

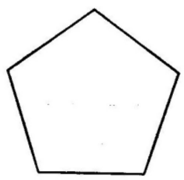
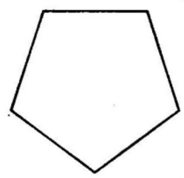


6.2 (i) State whether each of the following polygons is a regular polygon or not.
(ii) Give each polygon its usual name.

a)  **b)**  **c)** 

6.3 **a)** Which of the following polygons is not congruent to any of the others?
b) Give each polygon its usual name.

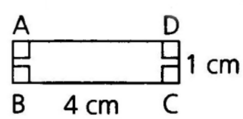
(i)  (ii)  (iii) 

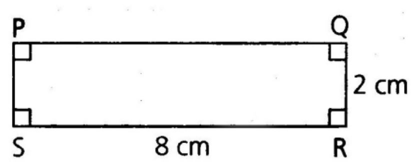
(iv)  (v) 

6.4 For each of the following statements:
(i) Say whether the statement is true or false
(ii) Give reasons for your answer in (i).

a) If a pentagon has its sides equal in length to the corresponding sides of a second pentagon, then the pentagons are congruent.

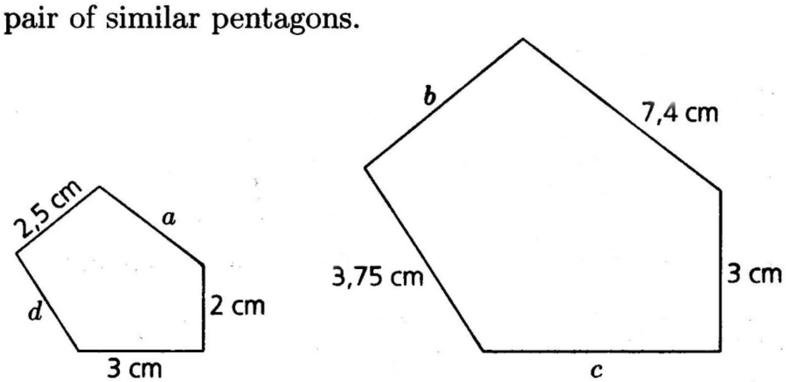
b) ABCD is similar to PQRS.





c) If the sides of a quadrilateral are equal to the corresponding sides of a second quadrilateral, the two quadrilaterals are congruent.

6.5 Calculate the lengths of the unknown sides (a, b, c and d) in the following pair of similar pentagons.



GEOMETRY: Answers to exercises

Exercise 1

1.1 $\hat{R}_3 = 180° - 70° - 110°$ (adj.∠'s st line)
$\hat{R}_2 = 70°$ (vert. opp)
$\hat{R} = 110°$ (adj. ∠'s st. line) or (vert. opp)
$\hat{S}_4 = 70°$ (corres.∠'s; AB//CD)
$\hat{S}_1 = \hat{R}_3 = 110°$ (alt.∠'s; AB//CD) etc....
+ many other options.

1.2 $\hat{S}_3 = 70°$ (alt.∠'s; UE//ZF)
$\hat{M}_1 = 80°$ (alt.∠'s; UE//ZF)
$\hat{M}_3 = \hat{S}_3 = 70°$ (corres.∠'s; TS//RM) etc...

1.3 a) $\hat{W}_2 = 30°$ (vert. opp.)
NOT // as this would need $\hat{V}_2 = \hat{W}_2$ .. corres.∠'s
b) $\hat{K}_2 + \hat{V}_1 = 170°$ NOT //;
need $\hat{K}_2 + \hat{V}_1 = 180°$ ... coint.∠'s
c) $\hat{P}_4 = 130°$ (vert. opp.)
$\hat{Q}_1 + \hat{P}_4 = 180°$
$\therefore //$ ... coint.∠'s supp.

1.4 $\hat{F} = x + 30°$ (corres.∠'s; AB//CD)
$\hat{E} = x$ (vert. opp.)
} in ΔEFG
$\therefore \hat{F} + \hat{E} + \hat{G} = 180°$
$\Rightarrow (x + 30°) + x + 3x = 180°$ (∠sum Δ)
$\Rightarrow x = 30°$

Exercise 2

2.1 $x = A\hat{C}B$ (isos Δ) $= 180° - 113°$ (adj.∠'s st.line) $= 67°$
$y = 180° - 2.67°$ (∠ sum Δ) $= 46°$ [*or* $y = 113° - 67°$ (ext.∠ Δ)]

2.2 $x = 66°$ (alt.∠'s; // lines)
$y = 44°$ (corres.∠'s; // lines)
$z = 180° - (44° + 66°)$ (∠ sum Δ *or* adj.∠'s st. line)
$= 70°$

2.3 $x = 180° - 125°$ (adj.∠'s st.line) $= 55°$
$\therefore \hat{A} = 55°$ (isos Δ), $\hat{D} = 80°$
$y = 2.55°$ (ext.∠ Δ) $= 110°$
$z = 180° - (55° + 80°)$ (∠ sum Δ) $= 45°$

2.4 $x + x + 64° = 180°$ (isos Δ; ∠ sum Δ) $\Rightarrow x = 58°$
$y + y = 58°$ (isos Δ; ext.∠ Δ) $\Rightarrow y = 29°$

2.5 $x = 70°$ (alt. ∠'s; AC//DE)
$y + y = 70°$ (isos Δ; ext.∠ Δ) $\Rightarrow y = 35°$

2.6 $x = 180° - (50° + 70°)$ (vert. opp.; ∠ sum Δ) $= 60°$
$y = 40° + 60°$ (ext.∠ Δ) $= 100°$
$z = 180° - (20° + 100°)$ (∠ sum Δ) $= 60°$

2.7 $A + 80° = 140°$ (ext.∠ Δ) $\Rightarrow A = 60°$
$x = 180° - 60°$ (coint.∠'s; AB//EF) $= 120°$
$80 + y = x + 10$ (ext.∠ Δ) $\Rightarrow y = 50°$
[*or* $x + y = 100°$ (alt.∠'s; DE//AB) $\Rightarrow x = 40°$]

2.8 $C + 20° = 80°$ (ext.∠ Δ) $\Rightarrow C = 60° = y$ (alt.∠'s; DE//BC)
$x + C = 100°$ (ext.∠ Δ) $\Rightarrow x = 40°$

2.9 $130° = y + 100°$ (ext.∠ Δ; adj.∠'s st line) $\Rightarrow y = 30°$
$x + y = 100°$ (ext.∠ Δ) $\Rightarrow x = 70°$

2.10 $B\hat{A}C = 60°$ (∠ sum Δ) $= x$ (alt.∠'s; CE//BA)
$E\hat{C}D = 360° - (x + 70° + 150°)$ (∠'s at a pt) $= 80°$

2.11 $50° + 100° + \hat{A}_1 = 180° = x + 80° + \hat{A}_2$ (∠ sum Δ) $\Rightarrow x = 70°$
$y + x + 100° = 180°$ (vert.opp.; ∠ sum Δ) $\Rightarrow y = 10°$

2.12 $x = 30°$ (corres.∠'s; BE//CD)
$A\hat{B}C + B\hat{C}A = 100°$ (∠ sum Δ) $\Rightarrow A\hat{B}C = 50°$ (isos Δ)
$E\hat{B}C = x = 30°$ (alt.∠'s; AE//BC)
$\therefore y = 50° - 30° = 20°$

2.13 $5x + x = 180°$ (adj.∠'s at line) $\Rightarrow x = 30°$
$2x + y = 180°$ (∠ sum Δ) $\Rightarrow y = 120°$

2.14 $5x + 4x = 180°$ (coint.∠'s; CD//AB) $\Rightarrow x = 20°$
$x + 4x + y = 180°$ (∠ sum Δ) $\Rightarrow y = 80°$

2.15 $(3a - 20°) + a = 180°$ (coint.∠'s; FG//DE) $\Rightarrow a = 50°$

$x = 50°$ (corres.∠'s; AB//DC)

$x + y + a = 180°$ (adj.∠'s st line) $\Rightarrow y = 80°$

Exercise 3

3.1 $x = 20$ 3.2 $x = 12; y = 13$ 3.3 $x = 25$ 3.4 $x = 9$ 3.5 Area $= 1386$

3.6 $x = 12, 88$ 3.7 $x = 15; y = 12$ 3.8 $x = 19, 80$ 3.9 $x = 9, 90$ 3.10 $x = 14.42$

Exercise 4

4.1 ΔABC ≡ ΔRPQ (SAS) 4.2 ΔABC ≡ ΔYXZ (SAS) 4.3 ΔABC ≡ ΔLMK (RHS)

4.4 ≢ 4.5 ΔABC ≡ ΔHJG (RHS) 4.6 ΔABC ≡ ΔMNL (SAA)

4.7 ≢ 4.8 ΔABC ≡ ΔWXY (SSS) 4.9 ≢

4.10 $\Delta ABC \equiv \begin{cases} \Delta DQP \\ \Delta QDP \end{cases}$ (SAA) 4.11 ΔABC ≡ ΔEBD (SAA) 4.12 ΔABC ≡ ΔTSC (SAS)

Exercise 6

6.1 (a) (i) convex (ii) hexagon (b) (i) concave (ii) octagon (c) (i) convex (ii) quadrilateral

6.2 (a) (i) yes (ii) decagon (b) (i) yes (ii) hexagon (c) (i) no (ii) hexagon

6.3 (a) (i) (ii) (b) all pentagons

6.4 (a) (i) true (ii) SSSSS (b) (i) true (ii) equiangular (c) (i) true (ii) SSSS